SIMULATION OF A HYDROSTATIC TRANSMISSION UNDERGOING CYCLIC INERTIAL TESTING

One of the most important implementations of mathematical models is in simulation, because it is relatively inexpensive to determine the suitability of a given system design before even one piece of hardware is acquired and before prototype systems are assembled. Furthermore, competing designs and hardware choices can be evaluated and compared. To illustrate the steps in conducting a simulation, a hydrostatic transmission has been modeled using the proposed Type 1 models for the pump and motor. Simulation will be used to study the dynamic details of the transmission during cyclic life testing. The hydrostatic transmission model, as simulated, is shown in Figure 1.

VARIABLE AND PARAMETER NOMENCLATURE FOR THE LINEARIZED HYDROSTATIC TRANSMISSION SIMULATION

Variables:
$D_P(t)$ = Pump displacement, $in^3/radian$
Q_{IP} = Ideal pump output flow, in^3/sec
Q_{LP} = Pump internal leakage flow, in^3/sec
Q_{aP} = Pump actual output flow, in^3/sec
Q_{aM} = Motor actual flow, in^3/sec
Q_{LM} = Motor internal leakage flow, in^3/sec
Q_{IM} = Ideal motor input flow, in^3/sec
T_{iP} = Pump input torque, *in-lb*
T_{IP} = Pump ideal torque, *in-lb*
T_{oM} = Motor output torque, *in-lb*
T_{IM} = Motor ideal torque, *in-lb*
ω_P = Pump shaft input speed, *rad/sec*
ω_M = Motor shaft output speed, *rad/sec*
Parameters:
D_M = Motor displacement, $in^3/radian$
R_{FWP} = Pump friction coefficient *in-lb/rad/sec*
R_{FWM} = Motor friction coefficient *in-lb/rad/sec*
R_{LP} = Pump leakage resistance, $psi/in^3/sec$
R_{LM} = Motor leakage resistance, $psi/in^3/sec$
C_H = Hydraulic capacitance, in^5/lb
J_L = Load polar moment inertia, $in\text{-}lb\text{-}sec^2$

TABLE 1 Listing of the variable names and units of measure for variables and parameters for the simulation of a linear model hydrostatic transmission.

In this simulation it is desired to determine the dynamic reaction of the transmission differential pressure and motor output speed in a planned life test. The test plan has the pump and motor plumbed with about 6 feet of 1-inch ID tubing. The motor is to be loaded with a flywheel that is sized to require about 3 seconds to accelerate from zero to 2400 rpm at a transmission differential pressure of 2400 psi. There are to be no pressure relief valves, instead, pressure will be controlled by cycling the variable pump's displacement from maximum in one direction to maximum in the other direction in about 6 seconds. We are interested in learning how the transmission pressure varies as acceleration and deceleration progress through time, as well as motor's output speed. Since proper life testing requires reaching targeted pressures and targeted speeds, the ability for the test system to reach 2400 psid and 2400 rpm is to be determined. In short, will the planned test set up do the job? The simulation is intended to provide some answers.

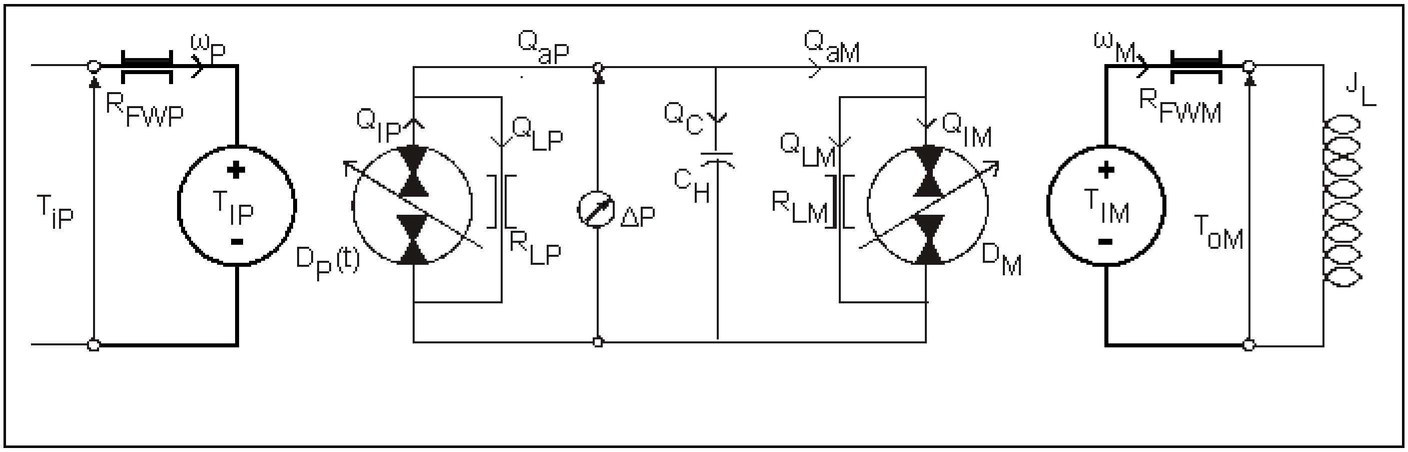

Figure 1 Analytical schematic of the linearized, Type 1, model of a hydrostatic transmission, showing algebraic symbology for the pertinent variables and parameters.

Figure 1 is an analytical schematic of the linearized hydrostatic transmission as constructed using the Type 1 pump and motor models. It shows that there are two shaft speeds, four torques, seven flows and seven parameters plus a time-varying pump displacement. Pump displacement is to be cycled in a manner that, hopefully, achieves targeted differential pressure and targeted motor output speed. Hydraulic capacitance is characterized with an electrical capacitor symbol, labeled C_H in Figure 1. The inertial load is characterized with a "curli-cue" symbol in the output circuit, similar to that for an electrical inductor (coil), of the motor and labeled J_L in the circuit schematic. The reason for borrowing electrical symbols is four-fold: 1) The electrical circuit laws of both Ohm and Kirchoff apply equally to linearized hydraulic circuits; 2) Borrowing of symbols is precedented, notably in the use of relay contacts for pneumatic logic circuits; 3) As early as 1950, Warren Wilson wrote of "hydraulic capacitance" in describing the effects of fluid compressibility; 4) It seems completely redundant to create a whole new circuit methodology for hydraulics when well-established electrical methods are so widely used, and if judiciously applied, are completely workable.

All the variables and parameters for the hydrostatic transmission are listed in TABLE

1 and evaluated in TABLE 2. In simulation it is necessary to know all of the parameters, one of the variables, and then the other variables can be calculated. Note further in TABLE 2 that the pump and motor have identical displacements, rated speeds and rated pressures. This is not at all necessary, it was merely done for convenience. Also, realize that the pump and motor are not of any particular type or manufacturer. They, if you will, are simply generic machines of quite conventional sizes and characteristics and performances.

TABLE 2 also has the calculated parameters, that is, the friction loss coefficients and the leakage resistances. The formulas for calculating them are given in the proposed math modeling standard, and, although not difficult to use, are not detailed here.

The simulation begins with the the writing of the defining circuit equations. Some of the mathematical steps are given here for those who are inclined to learn the methods, however, a complete understanding does require a knowledge of calculus, differential equations and transform methods for their solutions. Those who are not so inclined are urged to skip to the CONCLUSIONS section.

KEY PARAMETER VALUES FOR THE PUMP AND MOTOR FOR THE SIMULATION

PUMP
P_{Pr} = 2400 PSI
N_{Pr} = 2400 RPM = 251 radian/second
η_{VPr} = 90%
η_{MPr} = 90%
D_{Pr} = 0.5 in^3/radian
CALCULATED PUMP PARAMETRIC VALUES:
R_{fwP} = *** in-lb/(radian/sec)
$R_{\iota P}$ = 190.5 psi/(in^3/second)

MOTOR
P_{Mr} = 2400 PSI
N_{Mr} = 2400 RPM = 251 radian/second
η_{VMr} = 93%
η_{MMr} = 88%
D_{Mr} = 0.5 in^3/radian
CALCULATED MOTOR PARAMETRIC VALUES:
R_{fwM} = 0.574 in-lb/(radian/sec)
$R_{\iota M}$ = 273 psi/(in^3/second)

SYSTEM/CIRCUIT PARAMETERS
$C_H = 3.5x10^{-4}$ $inch^5$/lb (approx 6 ft of 1 inch ID tube plus pump and motor internal volumes, about 44 in^3, total pressurized volume, each side)
β = 125000 psi
J_L = 15 inch-lb-sec^2 (selected to require about 3 seconds acceleration time for the motor at 2400 psid)

TABLE 2 Key system parameter values that will be used in the simulation of the hydrostatic transmission that is undergoing cyclic life testing with an inertial load (flywheel) on the motor shaft.

In the instant example, there are three equations: 1) Summation of torques in the mechanical input circuit of the pump; 2) Summation of flows in the hydraulic circuit of the transmission; 3) Summation of torques for the mechanical output circuit of the motor.

Pump Input Torque Summation:

$$T_{iP} = R_{FWP}\omega_P + T_{IP} \tag{1}$$

Transmission Flow Summation:

$$Q_C = Q_{IP} - Q_{LP} - Q_{LM} - Q_{IM} = C_H\frac{dp}{dt} = D_P(t)\,\omega_P - \frac{p}{R_{LP}} - \frac{p}{R_{LM}} - D_M\omega_M \tag{2}$$

Motor Output Torque Summation:

$$T_{IM} = \omega_M R_{FWM} + J_L\frac{d\omega_M}{dt} = pD_M \tag{3}$$

It is necessary to solve these three equations, but with the knowledge that for simulation purposes, the pump speed, $\boldsymbol{\omega_P}$, is to be held perfectly constant, so that quantity is known. Equations (2) and (3) both contain the variables $\boldsymbol{p}$ (hydrostatic transmission differential pressure) and $\boldsymbol{\omega_M}$ (motor output speed), as well as their derivatives, ***dp/dt*** and $\boldsymbol{d\omega_M/dt}$, respectively. When the the several parametric values from TABLE 2 are inserted into Equations (2) and (3) and they are arranged into their canonical state variable forms, they become:

$$\frac{dp}{dt} = -25.5p - 1428\,\omega_M + 717000D_P(t) \tag{4}$$

and:

$$\frac{d\omega_M}{dt} = 0.333p - 0.383\,\omega_M \tag{5}$$

The equations must be solved simultaneously, for which there are several methods, but, because they are linear, Laplace Transforms will be used. Transformation produces the two transfer functions:

$$\frac{P(s)}{D_P(s)} = \frac{717000(s + 0.383)}{(s + 2.098)(s + 23.14\ )} \tag{6}$$

and:

$$\frac{\Omega_M(s)}{D_P(s)} = \frac{23898}{(s^2 + 25.2s + 48.5)} \tag{7}$$

Equations (6) and (7) are evaluated numerically by finding the Inverse Laplace Transforms, which can be accomplished by any of several methods. Certainly one wants to use a computer to do this and there are many programs available. I used my own program, ***IDAS***. The results include closed form analytical solutions (not shown) as well as graphical outputs, shown in Figure 2

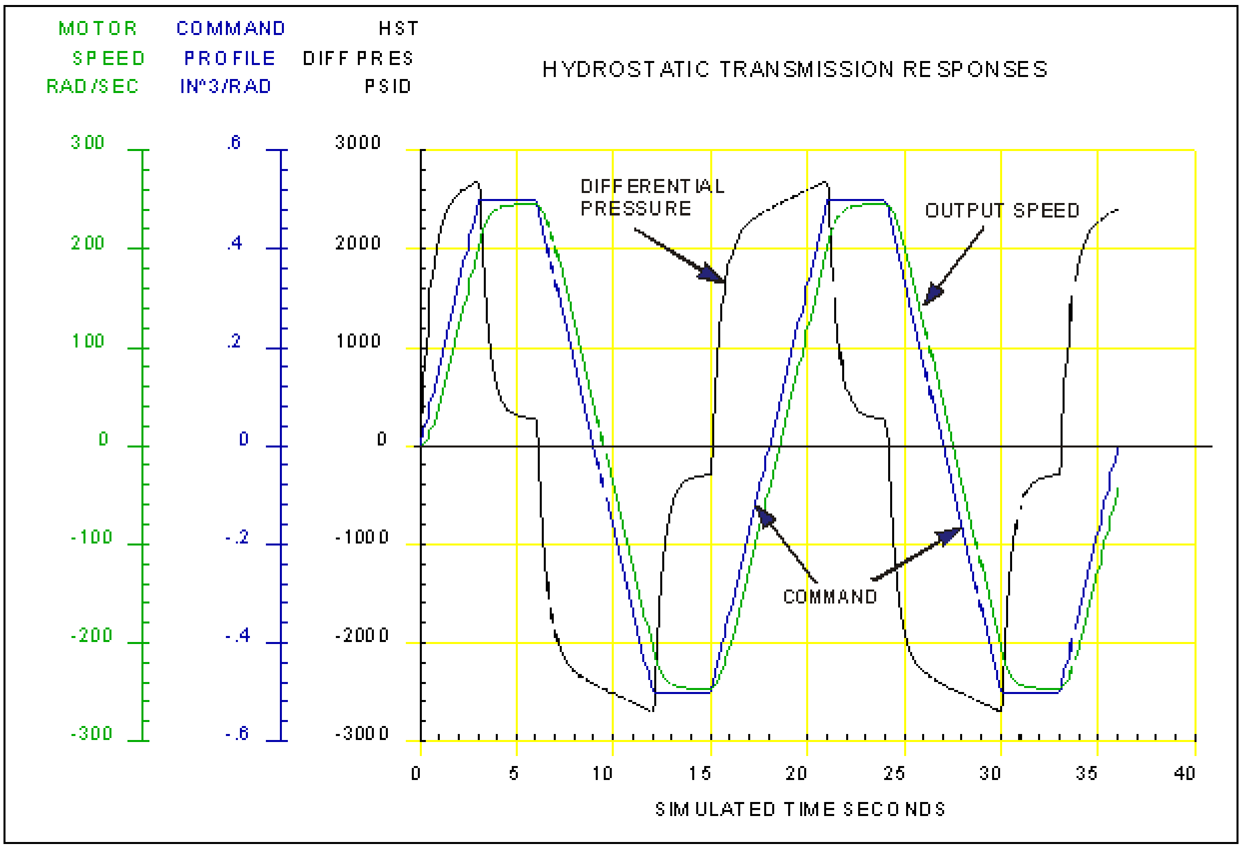

Figure 2 Pressure and speed responses of the linear hydrostatic transmission with a trapezoidal variation in pump displacement as a COMMAND input.

CONCLUSIONS

Recall, it was desired to simulate the testing of the hydrostatic transmission while the motor was loaded with a flywheel. The flywheel inertia was sized to require 3

seconds to accelerate the motor from 0 rpm to 2400 rpm (about 251 radians/second) while the pump and motor are operating at rated, 2400 psid, differential pressure. Cyclic operation was to be achieved by controlling the variable displacement of the pump from full displacement in one direction to full displacement in the other, and in the form of a trapezoidal waveshape, shown as the COMMAND signal in Figure 2.
Certainly, two of the questions to be asked of the simulation are: 1) Did the differential pressure reach the rated value of 2400 psi, and 2) Did the speed reach the targeted of value of 251 radians per second? The answers can be found in the time response graph. The pressure reached a peak value of almost 2700 psid, while the speed did not quite reach its target. Now, there are two more questions: 1) Will the higher than desired pressure and the lower than desired speed constitute a valid cyclic life test for the transmission, and if the answer to that question is NO, 2) What can be done to make the test valid? The answer to the first, as to the validity of the test, while being both legitimate and important, will be relegated to another forum. We would need much more information about the hardware to adequately answer it. The answer to the second, however, is much more intriguing. Let's assume that indeed, the test does not meet the requirements. What can be done?

There are a number of things that could be changed:
Pump displacement, but only decrease, because the maximum is 0.5 ***in^3/radian***
Decrease motor displacement; note that it is variable
Re-size the inertia of the flywheel
Increase pump shaft speed
Change the pump displacement profile shape, ie, reduce the rate of change.

All of these possibilities can be reasonably and approximately explored with the very simple hydrostatic transmission model introduced here. The entire study, above, took only about 2 hours of my time to see the graphical results. Writing this summary report took about 20 times than long. Each of the above "what if's" would require less than an hour. It is a small price to pay for the insights gained, and the results might surprise you, but they are more than can be covered here. But there can be little doubt about the value of math modeling and simulation.